PII : S0017–9310(96)00001–4

# The relationship between the Colburn and Silver methods of condenser design

D. R. WEBB

University of Manchester Institute of Science and Technology, PO Box 88, Manchester,
M60 1QD, U.K.

and

M. FAHRNER and R. SCHWAAB

Institut für Technische Thermodynamik und Thermische Verfahrenstechnik, Universität Stuttgart,
Stuttgart, Germany

*(Received 2 August 1995 and in final form 6 December 1995)*

**Abstract**—The most widely applied method for the design of mixed vapour condensers is the approximate method of Silver [1, *Trans. Inst. Chem. Engng* **25**, 30–42 (1947)] as modified by Bell and Ghaly [2, *AIChE Symp. Ser.* **69**, 72–79 (1972)]. However the older, and physically more realistic method of Colburn and Hougen [3, *Ind. Engng Chem.* **26**, 1178–1182 (1934)] and Colburn and Drew [4, *Trans. AIChemE* **33**, 197–215 (1937)] is often applied to the case of binary condensation. The relationship between these two methods is treated in a more complete and useful manner than in previous work. There has been an assumption that the two methods are equivalent at Lewis numbers near unity, but the approximate method may then be unsafe in design by up to 50%. The predictions are even less safe when the Lewis number is greater than unity, and cases are reported where the methods show discrepancies as large as 150% in gas film heat transfer coefficients. Discrepancies of similar magnitude are also found when the Lewis number is much less than unity, but the approximate method is then safe. The improved understanding that is reached leads to a new correction factor allowing the approximate method to be brought into good agreement with the Colburn approach in a self-consistent manner. The work is validated by comparison with the experimental data of Lehr [5, Doktor Dissertation Wärme und Stoffübergang bei der Kondensation von Dämpfen aus einem Gemisch mit einem Inertgas, Technische Universität, Hannover (1972)]. Copyright © 1996 Elsevier Science Ltd.

## INTRODUCTION

This paper makes a direct comparison of the popular Silver–Bell method, termed the *equilibrium* method, with the *film* methods due to Colburn and Hougen [3], Colburn and Drew [4] and Ackermann [6]. It is therefore of direct industrial relevance showing the error in the calculation of the gas side heat transfer coefficient. Both methods have long been available and have been compared before, most notably by McNaught, [9, 10]. The present work extends that of McNaught to give more physical insight into the influence of the important factors, temperature difference in the gas phase and Lewis number, so that better judgement can be exercised to decide whether the Silver method is acceptable.

All previous published work shows the superiority of the film model in predicting condensation of mixed vapours, Webb [8] and McNaught [10], with condensation rates predicted within 15% at worst. The Colburn method is a suitable reference for assessment of the Silver approach. Both methods may be applied to industrial design calculations, but the approximate method is preferred because it makes lesser demands for physical property data and does not require a full analysis of the composition of the mixture. The

present work agrees with McNaught on the magnitude of the error incurred in using the equilibrium approach, but shows cases where gas side heat transfer coefficients are under or over-estimated by up to 150%, even when his correction factor [9] is included.

A new way of correcting the equilibrium approach is suggested which allows the two methods to be brought into good agreement for the case of binary mixtures, without solving the film model equations, and this is demonstrated based on the data of Lehr [5]. His data are particularly suitable for this purpose because they have been taken in a simple geometry, condensation inside a single vertical tube without significant vapour shear, and show the effect of downstream length. Of the data considered by McNaught, it is the only set which presents the measured gas side rather than overall heat transfer coefficients, and is therefore more reliable. Significantly these are the data that are least well predicted by McNaught. The present work is an indication that considerable design errors arise in the application of the Silver method.

The widespread use of the approximate method in industrial practice and its apparent success is a reflection that many industrial condensers may be significantly over-surfaced.

## NOMENCLATURE

$A$    area
$\tilde{c}$    molar concentration
$\tilde{c}_p$    molar heat capacity
$\Delta \tilde{h}_v$    molar latent heat
$J$    molar diffusive flux
$\tilde{M}$    molar mass
$\dot{N}$    molar flowrate
$n$    number of components
$\dot{n}$    molar flux of condensation
$p$    pressure
$\dot{Q}$    heat transfer rate
$\dot{q}$    heat flux
$R_f$    dirt resistance
$r_i$    ratio of fluxes, $\dot{n}_i/\dot{n}_t$
$s$    coordinate direction
$s_f, s_t$    thickness of mass and heat transfer films
$T$    temperature
$U_{wo}$    overall heat transfer coefficient based on outside area
$\tilde{x}$    liquid mole fraction
$\tilde{y}$    vapour mole fraction
$\tilde{z}$    overall mole fraction
$Z$    $\dot{q}_g/\dot{q}_t$ by equilibrium and film methods.

Greek symbols

$\alpha$    heat transfer coefficient
$\beta_{ij}$    mass transfer coefficient for the $ij$ pair
$\Delta$    finite change of quantity
$\delta_{ij}$    binary diffusivity
$\varepsilon$    factor $\Sigma \dot{n}_i \tilde{c}_{pi}/\alpha_g$, ratio of sensible to conductive heat flux
$\theta$    molar vapour fraction
$\lambda$    thermal conductivity
$\xi, \xi_t$    mass and thermal rate factors
$\phi$    factor for binary mixture.

Dimensionless groups

$Le$    Lewis number
$Pr$    Prandtl number
$Sc$    Schmidt number.

Subscripts

bub    bubble point
c    relating to condensate
dew    Dew point
eff    effective
eq    relating to equilibrium method
f    denoting film average ( flux in $\tilde{c}_{pf}$ and $\Delta \tilde{h}_{vf}$)
f    relating to film method
g    relating to gas phase
$i,j$    indices giving component number
o    relating to coolant
sat    saturated
s    relating to interface
t    total, as in $\dot{n}_t$
t    thermal layer as in $s_t$ and $\xi_t$
w    relating to wall
wi    relating to inside wall
wm    relating to mean wall
wo    relating to outside wall as in $U_{wo}$.

Superscripts

o    modified variables in equations (60)–(63)
*    saturation quantity.

Overstrikes

$\sim$    molar quantity as in $\tilde{x}$
$\cdot$    flow rate as in $\dot{M}$.

## THEORY

The film and equilibrium methods are extensively documented and only sufficient of the theory is given to make the paper readable. Fuller descriptions have been presented by Webb and McNaught [7] and Webb [8].

*Mass transfer relations*

The methods differ in their treatment of the gas side. The approximate method does not consider the mass transfer process directly, as now described for the film method. Condensation of binary mixtures is treated in this work and Fig. 1 shows the underlying physical situation on which the film models of condensation are built.

The gas is a saturated or superheated mixture of $n$ species which may contain noncondensing gases as well as vapours. It is separated from the coolant by a solid wall on which condensate flows as a continuous film. Hypothetical, one-dimensional, steady, laminar films are imagined adjacent to the condensate surface where all gas side resistance to heat and mass transfer lies. Mass transfer is described by equation (1), and for a mixture of $n$ components the $n$ independent fluxes of condensation, $\dot{n}_i$, are given by,

$$\dot{n}_i = J_i + \tilde{y}_i \dot{n}_t, \quad i = 1 \text{ to } n$$

$$\begin{bmatrix} \text{molar} \\ \text{flux} \end{bmatrix} = \begin{bmatrix} \text{diffusive} \\ \text{flux} \end{bmatrix} + \begin{bmatrix} \text{convective} \\ \text{molar flux} \end{bmatrix}. \quad (1)$$

The condensation rates, $\dot{n}_i$, the number of moles which cross unit surface in unit time, are needed in design, and equation (1) shows a diffusive and a convective contribution to these fluxes. The particular
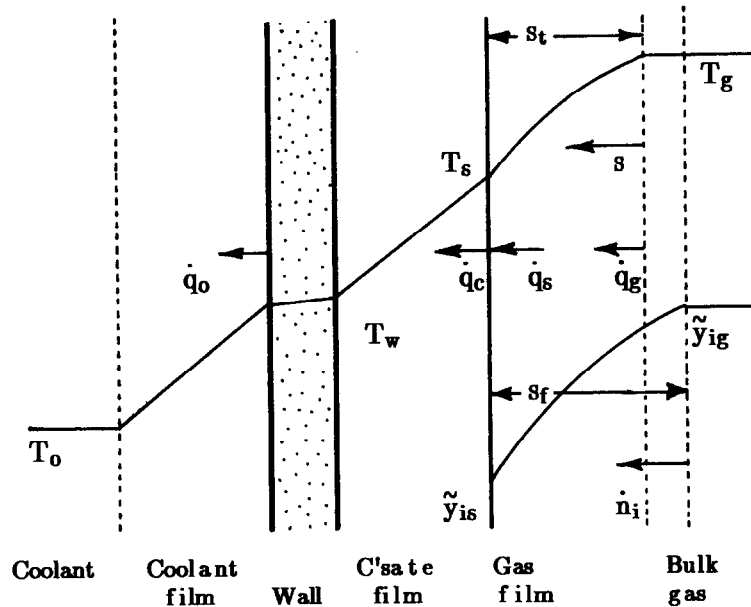
Fig. 1. Film model of condensation.

conditions of the film model, steady-state and one-dimensional, ensure that the $\dot{n}_i$ are constant across the film. Fick's law of diffusion allows the evaluation of the $J_i$, the diffusive fluxes, and therefore the prediction of mass transfer effects in condensation. The term $\tilde{y}_i \dot{n}_t$ is the bulk flow or convective contribution to the condensation rate, induced by the removal of heat to the coolant.

With two components, which may be two vapours or a vapour and a noncondensing gas, $(n = 2, \tilde{y}_1 < 1)$, Fick's Law gives $J_1$ as,

$$J_1 = -\tilde{c}\delta_{12}\frac{d\tilde{y}_1}{ds} \qquad (2)$$

where $\tilde{c}$ is the molar concentration. Equation (2) is used to solve equation (1) with the boundary conditions of Fig. 1, to give two equivalent equations for the mass transfer rate,

$$\dot{n}_1 = \beta_{12}\frac{\phi}{(e^\phi - 1)}(\tilde{y}_{1g} - \tilde{y}_{1s}) + \tilde{y}_{1g}\dot{n}_t \qquad (3)$$

$$\dot{n}_t = \beta_{12}\ln\left(\frac{r_1 - \tilde{y}_{1s}}{r_1 - \tilde{y}_{1g}}\right) : r_1 = \dot{n}_1/\dot{n}_t \qquad (4)$$

$$\phi = \dot{n}_t/\beta_{12} \quad \text{where } \beta_{12} = \frac{\tilde{c}\delta_{12}}{s_f}. \qquad (5)$$

The binary mass transfer coefficient, $\beta_{12}$ is defined by equation (5) and includes the unknown layer thickness, $s_f$, which is obtained from experiment. Equation (3) shows, through the so called *rate factor*, $\xi = \phi/(e^\phi - 1)$, an effect of mass transfer rate on the mass transfer coefficient, and the following definition of the modified mass transfer coefficient, $\beta_{12}\bullet$, is used :

$$\beta_{12}\bullet = \beta_{12}\frac{\phi}{(e^\phi - 1)} = \beta_{12}\xi. \qquad (6)$$

Neither equation (3), nor equation (4) is a complete description of condensation, because only the diffusional part of equation (1) is specified. Another relation, known as a determinacy condition, is required and it leads, directly or indirectly, to the specification of $\dot{n}_t$. It is needed because there are $n$ independent fluxes, $\dot{n}_i$, but only $(n-1)$ independent diffusive fluxes, $J_i$. The condensation rate is not fully defined by diffusional processes.

*Heat transfer relations*

The energy equation, with terms which describe steady conduction and convection, equation (7), is used to obtain rates of heat transfer across the gas film in Fig. 1.

$$\dot{q}_s = -\lambda_g\frac{dT}{ds} + \Sigma\dot{n}_i\tilde{c}_{pif}(T - T_s)$$

$$\begin{bmatrix}\text{heat} \\ \text{flux}\end{bmatrix} = \begin{bmatrix}\text{conductive} \\ \text{flux}\end{bmatrix} + \begin{bmatrix}\text{convective} \\ \text{heat flux}\end{bmatrix}. \qquad (7)$$

Equation (7) is readily integrated to give the conductive flux at the interface, $\dot{q}_s$

$$\dot{q}_s = \alpha_g\frac{\varepsilon}{(e^\varepsilon - 1)}(T_g - T_s) + \alpha_g\varepsilon(T_g - T_s)$$

$$\dot{q}_s = \alpha_g\frac{\varepsilon e^\varepsilon}{(e^\varepsilon - 1)}(T_g - T_s) \qquad (8)$$

where

$$\varepsilon = \frac{\Sigma \dot{n}_i \tilde{c}_{pif}}{\alpha_g} = \frac{\dot{n}_t \tilde{c}_{pf}}{\alpha_g} \quad \text{and} \quad \alpha_g = \frac{\lambda_g}{s_t}. \quad (9)$$

The heat transfer coefficient, $\alpha_g$, is defined by equation (9) and incorporates, $s_t$, the hypothetical thermal film thickness. It is obtained empirically. Equation (8) shows, through the Ackermann [6] correction factor, $\xi_t = \varepsilon/(e^{\varepsilon}-1)$, an effect of mass transfer rate on the heat transfer coefficient. The following definition of the modified heat transfer coefficient, $\alpha_g \bullet$, is used :

$$\alpha_g \bullet = \alpha_g \frac{\varepsilon}{(e^{\varepsilon}-1)} = \alpha_g \xi_t. \quad (10)$$

The heat transfer coefficient, $\alpha_g$, is that for conductive transfer alone for, in the limit of $\varepsilon$ approaching zero, $\xi_t$ approaches 1 and $\alpha_g = \alpha_g \bullet$. The similar roles of $\phi$ in equation (6) and $\varepsilon$ in equation (10) are clear. Equation (8) leads also to the rate of gas phase cooling, $\dot{q}_g$, as heat fluxes, $\dot{q}_g$ and $\dot{q}_s$ in Fig. 1, differ by the sensible heat change over the gas film. The rate of gas cooling is less than the heat transfer rate at the condensate surface.

$$\dot{q}_g = \alpha_g \frac{\varepsilon}{(e^{\varepsilon}-1)}(T_g - T_s). \quad (11)$$

The key to calculating the rates of combined heat and mass transfer is the continuity of energy across the interface, which from Fig. 1 may be written, $\dot{q}_o = \dot{q}_c = \dot{q}_s + \dot{n}_t \Delta \tilde{h}_v$, if condensate sub-cooling is neglected. An overall liquid side heat transfer coefficient, $\alpha_{wo}$, is defined including condensate, wall and coolant film,

$$\alpha_{wo}(T_s - T_o) = \alpha_g \frac{\varepsilon e^{\varepsilon}}{(e^{\varepsilon}-1)}(T_g - T_s) + \dot{n}_t \Delta \tilde{h}_{vf} = \dot{q}_o \quad (12)$$

where

$$\frac{1}{\alpha_{wo} A_{wo}} = \frac{1}{\alpha_c A_{wo}} + \frac{s_w}{\lambda_w A_{wm}} + \frac{1}{\alpha_o A_{wi}}. \quad (13)$$

The latent heat in equation (12) is a flux averaged value of the latent heats of the components at the interfacial temperature. Equation (12) is the general form of determinacy condition and can be solved for $\dot{n}_t$,

$$\dot{n}_t = \alpha_{wo}(T_s - T_o)/\left\{\Delta \tilde{h}_{vf} + \frac{\tilde{c}_{pf}(T_g - T_s)}{\{1 - \exp(-\varepsilon)\}}\right\} \quad (14)$$

where

$$\Delta \tilde{h}_{vf} = \Sigma \dot{n}_i \Delta \tilde{h}_{vif}/\dot{n}_t \quad \tilde{c}_{pf} = \Sigma \dot{n}_i \tilde{c}_{pif}/\dot{n}_t.$$

The overall heat transfer coefficient, $U_{wo}$, follows from equations (11) and (12) which with dirt resistance, $R_f$, gives

$$\frac{1}{U_{wo}} = \frac{1}{\alpha_{wo}} + R_f + \frac{Z_f}{\alpha_g \bullet}, \quad (15)$$

where $Z_f$ is the ratio of gas-side to total heat fluxes, $\dot{q}_g/\dot{q}_o$. It is present because the conductive flux across the gas film is less than the conductive flux to the coolant.

In the equilibrium approach the detailed mass transfer equations are not used. However, a gas side resistance is included by assuming that the sensible heat change of the saturated gas mixture must be conducted across the gas film. Bell [2] states that this assumption is usually conservative, but this is untrue, as shown by McNaught and the present work. The overall heat transfer coefficient is calculated by equation (15), where $\alpha_{wo}$ and $R_f$ are defined exactly as in the film approach. The term reflecting the gas side resistance is replaced by $Z_{eq}/\alpha_{g,eff}$. Figure 2 shows how $Z_{eq}$ is determined within an interval of heat load.

Figure 2 shows a condensate film on a vertical wall, but the horizontal case might equally well have been chosen. $\Delta \dot{Q}$ is the rate of heat removal in the interval as given by the cooling curve, and $\Delta \dot{Q}_g$ is the corresponding rate of sensible cooling of the gas phase to maintain saturation. Then $Z_{eq}$ is defined as follows :

$$Z_{eq} = \underset{\Delta \dot{Q} \to 0}{\text{Limit}} \frac{\Delta \dot{Q}_g}{\Delta \dot{Q}} = \left\{\frac{\dot{N}_g \tilde{c}_{pg} \Delta T_g}{\Delta \dot{Q}}\right\}_{sat} = \frac{\dot{q}_g}{\dot{q}}. \quad (16)$$

Equation (16) shows $Z_{eq}$ proportional to the slope of the condensation curve. The proper heat transfer coefficient for use in equation (15) has been investigated. In early work it was taken as the gas film heat transfer coefficient, $\alpha_g$, for the geometry in question. However McNaught [9] found that better results are given when it is corrected by the Ackermann [6] factor, $\varepsilon = \tilde{c}_p \dot{n}_t/\alpha_g$, see equation (10).

$$\alpha_{g,eff} = \alpha_g \frac{\varepsilon}{(e^{\varepsilon}-1)} = \alpha_g \bullet < \alpha_g. \quad (17)$$

Price and Bell [11] proposed a two-phase enhancement factor to account for the surface wave effects, but Owen et al. [12] found that this was not beneficial in predicting their data. In the absence of a reliable study resolving this contradiction, such a modification is not recommended.

The method is approximate, because mass transfer effects are not rigorously included by the above approach, nor as will be shown later is it always safe in design relative to the Colburn approach. The above equations are a sufficient set for the determination of the local condensation rates at a point within a condenser by both the film and equilibrium methods. The following equation, which appears in Schwaab [13], is important because it allows $Z_{eq} = \dot{q}_g/\dot{q}$ to be obtained locally, i.e. at a single plane by the equilibrium method

$$Z_{eq} = 1/\left(1 + \frac{(1-\theta)\tilde{c}_{pc}}{\theta \tilde{c}_{pg}} + \frac{\Delta \tilde{h}_v(T_g)}{\tilde{y}_{2g} \tilde{c}_{pg}} \frac{d\tilde{y}_{1g}}{dT_g}\right), \quad (18)$$

where $\theta$ is the molar fraction as vapour.

$$\dot{q}_g = \frac{dQ_g}{dA} \simeq \frac{\Delta Q_g}{\Delta A}$$

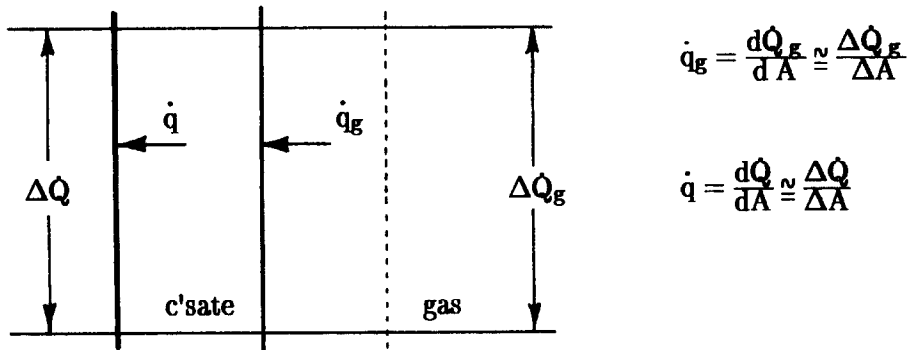$$\dot{q} = \frac{dQ}{dA} \simeq \frac{\Delta Q}{\Delta A}$$

Fig. 2. Heat fluxes in an interval.

## DIFFUSIONAL LIMITATION OF CONDENSATION RATES

The adverse effect of noncondensing gas on the rate of condensation of mixed vapours is well known. However, a diffusional limitation is also imposed when two vapours condense in the absence of a gas. The extent of this diffusional effect is now demonstrated.

### Two vapours

Figure 3, the phase or $(T, x)$, diagram for a binary mixture shows a saturated vapour at $B$, where $T_g = T_{dew}$, and coolant at $T_o$. The corresponding interfacial temperature must lie between $T_{dew}$ and $T_{bub}$ and is given by a tieline, such as FG, which defines the interfacial condensate and vapour compositions. The Colburn–Drew equation, equation (4), may be written in the following form, with the determinacy condition, $\tilde{x}_{1s} = \dot{n}_1/\dot{n}_t$, corresponding to unmixed condensate [8]:

$$\frac{(\tilde{x}_{1s} - \tilde{y}_{1g})}{(\tilde{x}_{1s} - \tilde{y}_{1s})} = \exp\left[-\frac{\dot{n}_t}{\beta_{12}}\right] = \exp[-\phi]. \quad (19)$$

In this case the tieline is determined by equation (19), which depends on the saturated interfacial conditions between condensate and vapour, $\tilde{x}_{1s}$ and $\tilde{y}_{1s}$. A second determinacy condition for mixed vapours is

$\tilde{x}_{1s}$ specified and FG is then defined by a bubble point calculation at an $\tilde{x}_{1s}$ defined by prior condensation. This corresponds to a fully mixed condensate. Real cases should lie between these two cases, which correspond to the limits of condensate side mass transfer.

For many cases of condensation, $\beta_{12}$, the mass transfer coefficient is small, $\beta_{12}\bullet(\tilde{y}_{1g}-\tilde{y}_{1s}) \ll \tilde{y}_{1g} \dot{n}_t$ in equation (1) and $\exp[-\phi]$ approaches zero in equation (19). Point $G$ lies close to $C$ and $\tilde{x}_{1s}$ tends to $\tilde{y}_{1g}$. The gas side transfer process is then the diffusion limited and $(\tilde{y}_{1g}-\tilde{y}_{1s})$ and $(T_g-T_s)$ become as large as possible. However, the bubble temperature of the gas mixture limits the mass transfer inhibition of condensation. A temperature driving force of $(T_{bub}-T_o)$ on the coolant side is the worst case.

The larger the mass transfer coefficient, the closer the interfacial composition lies to $H$. The gas side mass transfer resistance is small, $T_s \sim T_{dew}$, and the driving force $(T_{dew}-T_o)$ is available for transfer to the coolant. Real cases lie in between as shown by point $G$.

### Vapour and noncondensing gas

When a vapour is condensing in the presence of a noncondensing gas, the determinacy condition is $\dot{n}_1 = \dot{n}_t$ or $r_1 = 1$. Equation (4) holds with $r_1 = 1.0$, and the tieline FG can be drawn as in the previous case, Fig. 4.
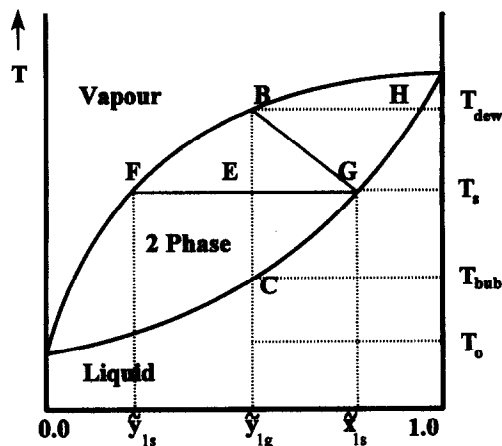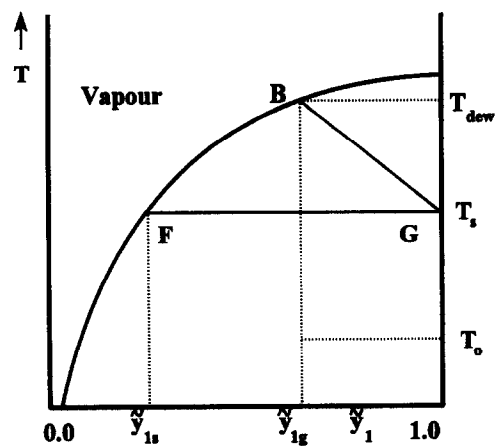


Fig. 3. Condensation of two vapours.



Fig. 4. Condensation of vapour from gas.

In strong contrast to two vapours, there is now no limit on the inhibition of mass transfer by diffusion. Thus, as $\beta_{12}$ becomes small, the driving force for heat transfer across the liquid side, $(T_s - T_o)$, also approaches zero. The difference between diffusional limitation in vapour mixtures and with a non-condensing gas is clear.

The condensing vapour sweeps noncondensing gas towards the interface, where it accumulates. At steady-state a balance is achieved, where the diffusion of gas away from the surface balances the flow due to condensation. With two vapours, the more volatile constituent behaves to some extent as a non-condensing species in that it accumulates at the interface, but the effect is smaller because it also condenses. Although these arguments are based on binaries, the same general behaviour is shown by multicomponent mixtures.

Diffusional effects are limited in vapour mixtures by the bubble temperature of the mixture which defines the lowest possible interfacial temperature and hence, the lowest heat transfer rate. There is no such limit with noncondensing gases.

## ERRORS INCURRED IN USING THE SILVER METHOD

Schwaab [13] carried out an extensive theoretical comparison of the Silver and Colburn methods for vertical downwards flow in a tube of 0.022 m diameter under the following conditions:

| vapours | water and $n$ heptane; |
|---|---|
| gases | helium, air and refrigerant 12; |
| pressure | 0.5–2.0 bar; |
| $\tilde{y}_{1g}$ and $\Delta T$ | 0.1–0.9 mole fraction and 10–30°C. |

The most important quantities in comparing the mixtures are the species present and the composition, because these influence the Lewis number $(Le = Sc/Pr)$ very strongly. The above mixtures have been chosen because they have a very large range of constituent molar masses and hence, of binary diffusivities and Lewis number. An error, $\varepsilon$, in effective gas film heat transfer coefficient is defined by equation (20), referred to Colburn because of its accepted reliability. The calculated errors are shown in the form $\varepsilon = \varepsilon'_{Le}$). The Silver method is unsafe when $\varepsilon > 0$. Values of $\varepsilon = 1$ and $\varepsilon = -0.5$ imply errors of $\pm 100\%$, unsafe and safe, respectively, in prediction of gas film heat transfer coefficient.

$$\varepsilon = (\alpha_g \bullet / Z_{eq}) / (\alpha_g \bullet / Z_f) - 1 \qquad (20)$$

The discrepancies in the local gas film heat transfer coefficients are very large. Thus, at a Lewis number of unity, where current thinking would have suggested that the methods should be in good agreement, the discrepancies show the approximate method to be unsafe in the prediction of gas film heat transfer coefficients, by up to 50%. The approximate method
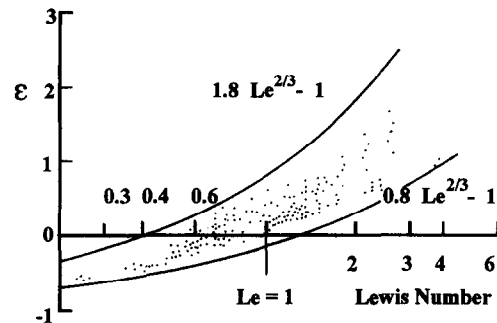


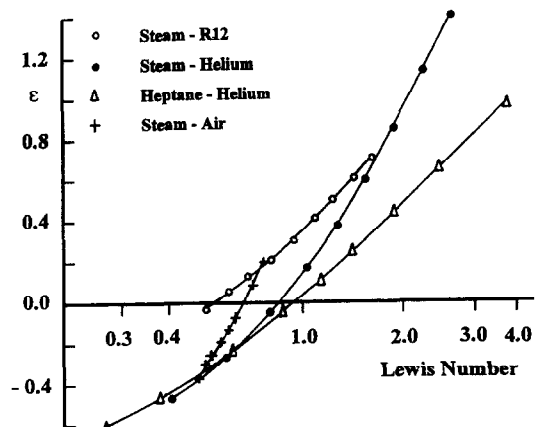Fig. 5. Discrepancy between Silver and Colburn.



Fig. 6. Discrepancy between Silver and Colburn for some mixtures.

shows errors of $\pm 150\%$, when the Lewis number is sufficiently different from unity. Limiting curves, of form $A\,Le^{2/3}$, as shown in Fig. 5, bracket the calculated errors, the form of the relationship being justified later. These curves represent safe limits.

Figure 6 shows some of the curves of individual mixtures, which in all cases correspond to compositions from 0.1 to 0.9 by mole fraction. Thus, a particular curve shows the effect of composition on the Lewis number at constant pressure and temperature difference. If the condensing curve for a particular design straddles the abscissa $\varepsilon = 0$, there will be a compensatory effect in the prediction of area requirement and this may account for the satisfactory behaviour of the Silver method in some cases. This is probably true of steam–air mixtures, where most data are available, because, as shown by the curve in Fig. 6, both positive and negative discrepancies are exhibited over a typical condensing range. It follows that the Silver method may become unacceptable by merely changing the range of composition shown in the condensation. Where a curve lies wholly above or below the abscissa in Fig. 6, no compensation will occur, and larger discrepancies will be realized.

There are considerations which tend to make the above errors somewhat less serious in practice. In design, the coolant side resistance may be of comparable size to that of the gas side and a conservative
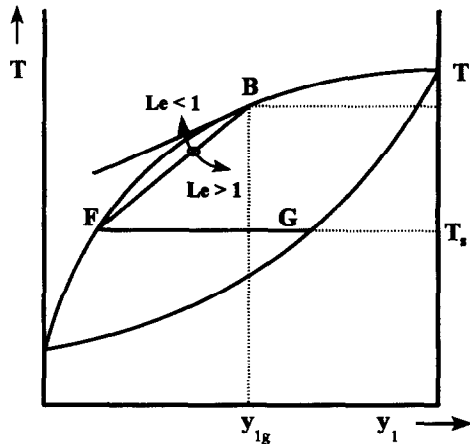
Fig. 7. The effect of Lewis number on $T_g$.

fouling factor may be allowed. The overall heat transfer coefficient may not, as a result, be strongly affected by the gas side. Nevertheless, the discrepancies are very significant, and the Colburn method is always preferable in design.

## THE RELATIONSHIP BETWEEN THE SILVER AND COLBURN METHODS

In the equilibrium method it is assumed that the gas remains saturated. Its temperature and composition will change along the dew line in Fig. 7, and locally the condensation path, $(dT_g/d\tilde{y}_{1g})$, will follow the tangent at $B$. In the film method, however, the gas temperature follows a path determined by the relative rates at which heat and mass transfer occur across the gas film. The description of this is provided by the Colburn–Edison equation [14], written as,

$$\frac{dT_g}{d\tilde{y}_{1g}} = \frac{(T_g - T_s)}{(\tilde{y}_{1g} - \tilde{y}_{1s})} \frac{\xi_t}{\xi} Le^{2/3}, \tag{21}$$

where $Le$ is the Lewis number $(Sc/Pr)$. It is necessary to understand the behaviour of the term

$$\frac{\xi_t}{\xi} = \frac{\varepsilon}{\phi} \frac{(e^\phi - 1)}{(e^\varepsilon - 1)}.$$

For small $\varepsilon$ and $\phi$ ($<0.1$), the term lies within 5% of unity. For larger $\varepsilon$ and $\phi$, it varies weakly with Lewis number and intensifies the effect of the Lewis number in determining the condensation path.

The influence of each factor in equation (21) can now be described. The first is considered in relation to the tieline FG, which defines the interfacial conditions of temperature and composition, $T_s$, $\tilde{x}_{1s}$ and $\tilde{y}_{1s}$, (Fig. 3). As the slope of chord BF it shows a tendency of the gas temperature to follow chord BF, in Fig. 7, and to become subcooled. The second factor, $\xi_t/\xi$, is neutral at low fluxes, but has a weak effect of increasing the power of Lewis number above 2/3 at high fluxes. The third factor shows the direct effect of Lewis number. With $Le > 1$, the gas has an increased

tendency to subcool, but with $Le < 1$ a smaller tendency, as shown in Fig. 7.

These trends are summarized. The film methods predict that during condensation the saturated vapours have an inherent tendency to subcool, even at $Le = 1$ and low transfer rates, because the condensation path is directed along the chord at $B$, defined by the interfacial tieline FG. This tendency is increased at $Le > 1$, but decreased at $Le < 1$. At high transfer rates the tendency with Lewis number is increased. Only when $Le$ is somewhat smaller than 1 is there a tendency to superheat, see Fig. 7, corresponding to the condensing path lying above the tangent at $B$.

It is now possible to make a simple comparison of the film and equilibrium methods, by comparison of the effective gas side resistance, $(Z/\alpha_g\bullet)$, as predicted by the two methods. This term may be expressed, using the equation of downstream change of gas temperature,

$$Z/\alpha_g\bullet = -\frac{\dot{N}_g \tilde{c}_{pg}}{\dot{q}\alpha_g\bullet}\left(\frac{dT_g}{dA}\right). \tag{22}$$

The comparison is made for the same gas mixture, fixed $\dot{N}_g\tilde{c}_{pg}$. As suggested by McNaught, both methods use the same heat transfer coefficient $\alpha_g\bullet$. However, the total heat flux, $\dot{q}$, usually differs because different condensate temperatures are assumed. In the equilibrium approach the saturation temperature of the gas is used, while with the film method it is the subcooled condensate temperature. Since the heat of subcooling is usually small, the difference is not important. Now it is apparent that both $d\tilde{y}_{1g}/dA$ and $\dot{q}$ are proportional to $\dot{n}_1$ and that equation (22) may be written,

$$\left(\frac{\alpha_g\bullet}{Z_{eq}}\right) = \left(\frac{\alpha_g\bullet}{Z_f}\right)\left(\frac{dT_g}{d\tilde{y}_{1g}}\right)_f \Bigg/ \left(\frac{dT_g}{d\tilde{y}_{1g}}\right)_{eq} \tag{23}$$

$$\left(\frac{\alpha_g\bullet}{Z_{eq}}\right) = \left(\frac{\alpha_g\bullet}{Z_f}\right)\frac{(T_g - T_s)}{(\tilde{y}_{1g} - \tilde{y}_{1s})} \frac{\xi_t}{\xi} Le^{2/3} \Bigg/ \left(\frac{dT_g}{d\tilde{y}_{1g}}\right)_{sat}. \tag{24}$$

Equations (23) and (24) explain the discrepancy between the film and equilibrium methods for binary mixtures. The effective gas side heat transfer coefficients are in the ratio shown by the slopes of the condensation paths in Fig. 7. This allows the relationship between the methods to be quantified.

When $(dT_g/d\tilde{y}_{1g})_f > (dT_g/d\tilde{y}_{1g})_{eq} = (dT_g/d\tilde{y}_{1g})_{sat}$ the Silver method is unsafe compared with the film method, as used in industrial design, showing a higher gas film heat transfer coefficient. This is the usual case, because, as seen in Fig. 7, a comparison is being made between the chord and the tangent at $B$. When $Le > 1$, the value of $dT_g/d\tilde{y}_{1g}$ is even greater and the Silver method still less safe in design. Conversely at $Le < 1$, the methods are in better agreement. If $Le$ is small enough the Silver method will become safe.
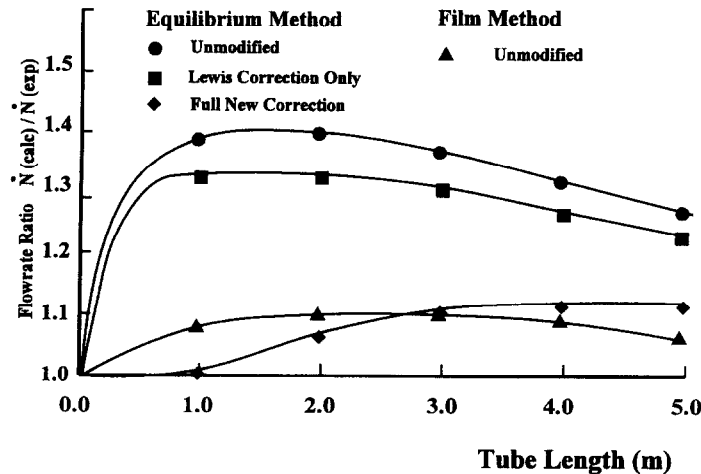
Fig. 8. Comparisons with experiment (benzene–nitrogen).

The features seen in Fig. 5 are now readily understood. It is the difference between the slopes of the tangent and the chord in Fig. 7 and their appearance in equation (24) that causes the approximate method to be unsafe at $Le = 1$. The Lewis number, through the factor $Le^{2/3}$, is responsible for the general shape of Fig. 5. The general variability is the effect of the flux ratio $\xi_t/\xi$ and liquid subcooling.

The role of an increased gas side temperature driving force is also clear, because it serves to increase the slope of the chord BF in Fig. 7. It is also important to recognize that the treatment of the equilibrium method here is differential. When the cooling curve is calculated at discrete intervals the condensation path follows a chord in Fig. 7, improving the agreement between film and equilibrium approaches.

The Silver approach is more commonly used in the industrial design of multicomponent condensers. With unspecified mixtures, where only a cooling curve is available, it must be used. Its advantage is that no further vapour–liquid equilibrium calculations are needed, irrespective of geometry. It has been corrected and improved by use of the more physically realistic film model, but it is shown above that substantial discrepancies between the methods can occur, even when all published correction factors are included. The success of the Silver method in industrial practice must be ascribed to the fact that industrial design is extremely conservative.

## A NEW CORRECTION FACTOR FOR THE SILVER METHOD

The experimental verification of the present work has been carried out by Fahrner [15], using the experimental data of Lehr [5], Figs. 8 and 9, which are for binary condensation of benzene and toluene, respectively, from nitrogen. The comparisons allow the assessment of the relative predictions of the film and equilibrium methods as described in this paper, and the improvements that are possible based on appli-

cation of equation (24), which provides a way of correcting the gas side heat transfer resistance to bring the equilibrium method into closer agreement with the film model.

$$(Z_t) = (Z_{eq})\frac{(T_g - T_s)}{(\hat{y}_{1g} - \hat{y}_{1s})}\frac{\xi_t}{\xi}Le^{2.3}\bigg/\left(\frac{dT_g}{d\hat{y}_{1g}}\right)_{sat}. \quad (25)$$

The data are taken for downwards flow of vapour and condensate in a single, vertical tube of 0.04 m diameter, and downstream length of 5 m, with measurements of condensation rate made at 1 m intervals. The wall temperature is constant and in consequence the unreliability of estimating the gas-side coefficient from an overall coefficient is eliminated. The data are taken under conditions of low vapour shear but this has been accounted for in the comparison. The equations of downstream change, which are given for the film method in Webb [8] are integrated by a fourth-order Runge–Kutta (RK) method.

Figures 8 and 9 are plots of the ratio of the predicted to experimental condensation rates as a function of condenser length by a number of different methods for the systems benzene–nitrogen and toluene–nitrogen, respectively.

In both cases, the film method predicts the condensation rate within 10–15%, which is typical of the agreement found in the literature, Webb [8] and McNaught [10]. The largest discrepancy between the methods is observed near the inlet. The equilibrium method is about 40% unsafe for benzene–nitrogen and 80% unsafe for toluene–nitrogen. Overall, as shown by the comparison of condensation rates over the full tube length of 5 m, the agreement is better and the margins of unsafe design are halved. The overall discrepancies seen between the theory and data reported in the present work are in good agreement with the values reported by McNaught [10]. The improved agreement of the overall condensation rates is a consequence of the variation of Lewis number with composition. As condensation proceeds in these
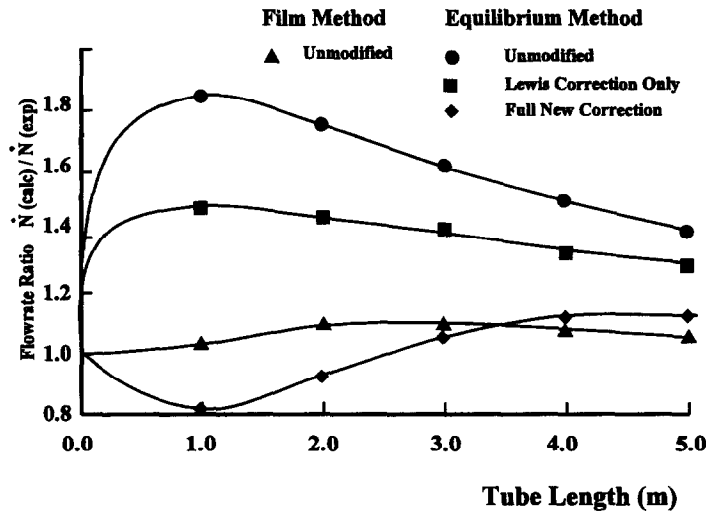
Fig. 9. Comparisons with experiment (toluene–nitrogen).

examples, the Lewis number falls from a value above unity to a value below. The compensatory mechanism described above, where the equilibrium method is initially unsafe, but later safer relative to the film method, leads to the observed improvement seen with downstream length.

Application of the correction factor derived from equation (25) brings about a marked improvement of the equilibrium method. A limited and very simple correction based on just the factor $Le^{2/3}$ is applied first, as shown in both Figs. 8 and 9. The discrepancy between the equilibrium and film methods is halved and the Lewis number is therefore responsible for about 50% of the discrepancy. The full correction factor shown in equation (25) is then applied. This has the effect of bringing the equilibrium method into very good agreement with the data. The final validation of the correction needs further work, but it looks very promising in this preliminary comparison.

The correction factor given in equation (25) provides a self-consistent correction to the equilibrium method because, as presented in this work, it does not require a film model solution to be carried out. The interfacial temperature, $T_s$, appropriate for the equilibrium method, follows from a knowledge of gas and condensate side heat transfer coefficients and the corresponding compositions, $\tilde{x}_{Is}$ and $\tilde{y}_{Is}$, are then uniquely determined for binary mixtures. The factors $\zeta_t$ and $\zeta$ may then be estimated from the condensation rate and the full correction factor follows. The procedure is iterative in that $T_s$ depends on the gas side heat transfer coefficient.

## CONCLUSIONS

(1) The relation between the equilibrium method of Silver, Bell and Ghaly and the film method of Colburn and co-workers is explained in a more complete way than in previous work, and some previous misconceptions are corrected. The two methods are

not in agreement at a Lewis number of unity, but at a value somewhat below, typically 0.6–0.8. At $Le = 1$ the equilibrium method is unsafe in predicting the gas film heat transfer coefficient by up to 50%. As Lewis number increases above unity the equilibrium method is increasingly unsafe. The equilibrium method appears to be good in many assessments against experimental data, because of a compensatory mechanism whereby variation of the Lewis number over the condensation range gives safe and unsafe predictions to an almost equal extent, but good predictions on average. The factors which lead to large errors are Lewis number very different from unity, large temperature driving force and high condensation rates, and a narrow condensation range as in a partial condenser.

(2) The most commonly used industrial design method has been shown to be very unsafe in many cases of mixed vapour condensation, even when the usual corrections for mass transfer are included. Since industry has accepted the predictions it would imply that many condensers are very over-surfaced. The examples quoted in this work, based on Lehr's experimental data, show errors of 80% in the local heat transfer coefficient and up to 40% in the overall coefficient, but in design cooling, dirt and fouling resistances would reduce this. The hypothetical cases examined show errors up to 150%, safe and unsafe. The conclusion reached here is derived from consideration of binary mixtures, but there is no reason to suppose that the situation is different for multicomponent condensers.

(3) A new correction factor has been proposed that brings the film and equilibrium methods into good agreement. This method has ben shown to work for binary mixtures where interfacial conditions are defined by temperature alone.

the Research Programme has been by the EPSRC and the following Companies, Hick Hargreaves and Company of Bolton, Lancashire and B-JAC, of Midlothian, USA.

## REFERENCES

1. L. Silver, Gas cooling with aqueous condensation, *Trans. Inst. Chem. Engng* **25**, 30–42 (1947).
2. K. J. Bell and M. A. Ghaly, An approximate generalised design method for multicomponent-partial condensers, *AIChE Symp. Ser.* **69**, 72–79 (1972).
3. A. P. Colburn and O. A. Hougen, Design of cooler condensers for mixtures of vapours with non-condensing gases, *Ind. Engng Chem.* **26**, 1178–1182 (1934).
4. A. P. Colburn and T. B. Drew, The condensation of mixed vapours, *Trans. AIChemE* **33**, 197–215 (1937).
5. G. Lehr, Wärme und Stoffübergang bei der Kondensation von Dämpfen aus einem Gemisch mit einem Inertgas, Doktor Dissertation, Technische Universität, Hannover (1972).
6. G. Ackermann, Combined heat and mass transfer in the same field at high temperature and partial pressure differences, *Forsch. Ingenieurwes.* **8**, 1–16 (1937).
7. D. R. Webb and J. M. McNaught, *Condensers, Developments in Heat Exchanger Technology.* Applied Science, Barking, U.K. (1980).
8. D. R. Webb, Multicomponent condensation, *Plenary Lecture, 9th Int. Heat Trans. Conf.*, Jerusalem, August (1990).
9. J. McNaught, Mass transfer correction terms in design methods for multi-component-partial condensers, *18th Natl. Heat Transfer Conf.*, San Diego, ASME-AIChE (1979).
10. J. McNaught, An assessment of design methods for condensation of vapours from a non-condensing gas, *ICHMT Seminar on Advances in Heat Exchangers*, Dubrovnik (1981).
11. B. C. Price and K. J. Bell, Design of binary vapour condensers using the Colburn–Drew equations, *AIChEJ Symp. Ser.* **8**, 267–272 (1974).
12. R. G. Owen, R. G. Sardesai and D. J. Pulling, Transfer coefficients for condensation in a vertical tube, Paper 80-HT-55, *19th Natl. Heat Transfer Conf*, ASME/AIChE, Orlando (1980).
13. R. Schwaab, Comparison of condenser design methods, Diplomarbeit University of Stuttgart/UMIST, Manchester (1993).
14. A. P. Colburn and A. G. Edison, Prevention of fog in cooler-condensers, *Ind. Engng Chem.* **33**, 457–458 (1941).
15. M. E. Fahrner, Development and experimental validation of condenser design methods, Diplomarbeit, University of Stuttgart/UMIST, Manchester (1994).